(6 pages) **Reg. No. : ....**

Code No. : 30009 E **Sub. Code : GMMA 61/ GMMC 61**

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Sixth Semester

Mathematics/Mathematics with C.A. – Main

COMPLEX ANALYSIS

(For those who joined in July 2012 – 2015)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 1 = 10$ marks)

Answer ALL the questions.

Choose the correct answer :

1. $(1-i)^4 =$

   (a) $4i$ (b) $-4i$

   (c) 4 (d) -4

2. Which of the following is a region?

(a) $\{Z : |Z - 2 + i| \le 1\}$ (b) $\{Z : |\text{Im } Z| > 1\}$

(c) $\{Z : \text{Re } Z > 1\}$ (d) $\{Z : |\text{Im } Z| \ge 1\}$

3. $\lim_{Z \to 2} \frac{Z^2 - 4}{Z - 2} =$

(a) 0 (b) 1

(c) 4 (d) ∞

4. If $f(z)$ and $\overline{f(z)}$ are analytic, then $f(z) =$

(a) 0 (b) constant

(c) $f(\bar{z})$ (d) $\overline{f(z)}$

5. $(i, 0, -1, \infty) =$

(a) $i+1$ (b) $i-1$

(c) $1-i$ (d) i

6. The invariant points of the transformation $W = \frac{1+2}{1-2}$ are

(a) 0, 1 (b) 1, –1

(c) $i, -i$ (d) 0, i

7. If C is the circle $|z| = 2$, then $\int_C \frac{\sin z}{\left(z - \frac{\pi}{2}\right)^2} dz =$

(a) $2\pi i$ (b) 4π

(c) $4\pi i$ (d) 0

8. $1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + ... =$

(a) $\sin z$ (b) $\cos z$

(c) e^z (d) e^{-z}

9. If $f(z) = \frac{e^z}{z^2}$, then $\text{Res}\{f(z);0\} =$

(a) 0 (b) –1

(c) 1 (d) ∞

10. The singular points of the function $f(z) = \frac{z-1}{z^2 - 5z + 6}$ are

(a) 1, 0 (b) 2, 3

(c) –2, –3 (d) –5, 6

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove : $|z_1 - z_2| \geq ||z_1| - |z_2||$.

Or

(b) Prove : $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$.

12. (a) Verify that whether the function $f(z) = \bar{z}$ is differentiable.

Or

(b) If $u(x, y) = x^4 - 6x^2y^2 + y^4$, find the analytic function $f(z) = u(x, y) + iu(x, y)$.

13. (a) Show that the transformation $W = \frac{5-4z}{4z-2}$ maps the unit circle $|z| = 1$ into a circle of radius unity and centre $\frac{-1}{2}$.

Or

(b) Find the bilinear transformation which maps the points $-1, 1, \infty$ respectively onto $-i, -1, i$.

14. (a) Prove : $\int_C \bar{z}^{-2}\, dz = 0$ if C is the unit circle $|z| = 1$.

Or

(b) Evaluate $\int_C \frac{z dz}{z^2 - 1}$ where C is the positively oriented circle $|z| = 2$.

15. (a) Expand $\frac{-1}{(z-1)(z-2)}$ as a power series in z in the region $1 < |z| < 2$.

Or

(b) Evaluate : $\int_C \frac{3z^2 + z - 1}{(z^2-1)(z-3)} dz$ where C is the circle $|z| = 2$.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that z_1 and z_2 are inverse points with respect to a circle $z\bar{z} + \bar{\alpha} z + \alpha \bar{z} + \beta = 0$ if and only if $z_1 \bar{z_2} + \bar{\alpha} z_1 + \alpha \bar{z_2} + \beta = 0$.

Or

(b) Find the point $Q = (x_1, x_2, x_3)$ on the sphere S that represents the complex number $z = x + iy$.

17. (a) Derive the Cauchy-Riemann equations in polar coordinates.

Or

(b) Find the analytic function $f(z)=u+iv$ if $u+v=\dfrac{\sin 2x}{\cosh 2y-\cos 2x}$.

18. (a) Find the image of the circle $|z-3i|=3$ under the map $W=\dfrac{1}{z}$.

Or

(b) Prove that a bilinear transformation $W=\dfrac{az+b}{cz+d}$ where $ad-bc\neq 0$ maps the real axis into itself if and only if a, b, c, d are real.

19. (a) State and prove Cauchy's theorem.

Or

(b) Expand ze^{2z} in a Taylor's series about $z=-1$ and determine the region of convergence.

20. (a) State and prove Laurent's theorem.

Or

(b) Evaluate : $\displaystyle\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$.

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